

UNIVERSITY CALCULUS

EARLY TRANSCENDENTALS



HASS • HEIL • BOGACKI • WEIR • THOMAS

FOURTH EDITION

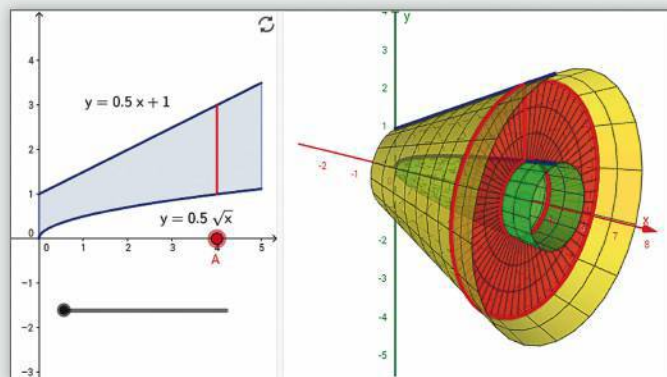
MyLab Math for University Calculus, 4e

(access code required)

MyLab™ Math is the teaching and learning platform that empowers instructors to reach every student. By combining trusted author content with digital tools and a flexible platform, MyLab Math for *University Calculus, 4e* personalizes the learning experience and improves results for each student.

Interactive Figures

A full suite of Interactive Figures was added to illustrate key concepts and allow manipulation. Designed in the freely available GeoGebra software, these figures can be used in lecture as well as by students independently.



Question Help

A force of 18 N will stretch a rubber band 12 cm (0.12 m). Assuming that Hooke's law applies, how far will a 6-N force stretch the rubber band? How much work does it take to stretch the rubber band this far?

How far will a 6-N force stretch the rubber band?

m
(Simplify your answer.)

Set up the integral that gives the work required, in joules.

$W = \int_0^{0.04} 150x \, dx$

How much work does it take to stretch the rubber band given a 6-N force?

J
(Simplify your answer.)

Questions that Deepen Understanding

MyLab Math includes a variety of question types designed to help students succeed in the course. In Setup & Solve questions, students show how they set up a problem as well as the solution, better mirroring what is required on tests. Additional Conceptual Questions were written by faculty at Cornell University to support deeper, theoretical understanding of the key concepts in calculus.

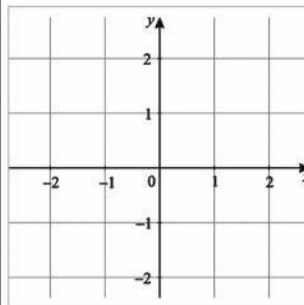
Learning Catalytics

Now included in all MyLab Math courses, this student response tool uses smartphones, tablets, or laptops to engage students in more interactive tasks and thinking during lecture. Learning Catalytics™ fosters student engagement and peer-to-peer learning with real-time analytics.

10:41 AM learningcatalytics.com

sketch question

Sketch a graph of a function f that satisfies the following conditions: $\lim_{x \rightarrow -\infty} f(x) = 2$, $\lim_{x \rightarrow 0^-} f(x) = -\infty$, $\lim_{x \rightarrow 0^+} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = 0$.



* Clear sketch Submit response

10:45 AM learningcatalytics.com

multiple choice question

True or False. As x increases to 100, $f(x) = \frac{1}{x}$ gets closer and closer to 0, so the limit as x goes to 100 of $f(x)$ is 0. Be prepared to justify your answer.

A.

B.

[Send a message to the instructor](#)

[Join another session](#)

UNIVERSITY CALCULUS EARLY TRANSCENDENTALS

Fourth Edition

Joel Hass

University of California, Davis

Christopher Heil

Georgia Institute of Technology

Przemyslaw Bogacki

Old Dominion University

Maurice D. Weir

Naval Postgraduate School

George B. Thomas, Jr.

Massachusetts Institute of Technology



Director, Portfolio Management: Deirdre Lynch
Executive Editor: Jeff Weidenaar
Editorial Assistant: Jonathan Krebs
Content Producer: Rachel S. Reeve
Managing Producer: Scott Disanno
Producer: Shannon Bushee
Manager, Courseware QA: Mary Durnwald
Manager, Content Development: Kristina Evans
Product Marketing Manager: Emily Ockay
Product Marketing Assistant: Shannon McCormack
Field Marketing Manager: Evan St. Cyr
Senior Author Support/Technology Specialist: Joe Vetere
Manager, Rights and Permissions: Gina Cheselka
Text and Cover Design: Pearson CSC
Full Service Vendor: Pearson CSC
Full Service Project Management: Julie Kidd, Pearson CSC
Manufacturing Buyer: Carol Melville, LSC Communications

Cover Image: Steve Mann/123RF

Copyright © 2020, 2016, 2012 by Pearson Education, Inc. 221 River Street, Hoboken, NJ 07030. All Rights Reserved. Printed in the United States of America. This publication is protected by copyright, and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise. For information regarding permissions, request forms and the appropriate contacts within the Pearson Education Global Rights & Permissions department, please visit www.pearsoned.com/permissions/.

Attributions of third-party content appear on page C-1, which constitutes an extension of this copyright page.

PEARSON, ALWAYS LEARNING, and MYLAB are exclusive trademarks owned by Pearson Education, Inc. or its affiliates in the U.S. and/or other countries.

Unless otherwise indicated herein, any third-party trademarks that may appear in this work are the property of their respective owners and any references to third-party trademarks, logos or other trade dress are for demonstrative or descriptive purposes only. Such references are not intended to imply any sponsorship, endorsement, authorization, or promotion of Pearson's products by the owners of such marks, or any relationship between the owner and Pearson Education, Inc. or its affiliates, authors, licensees or distributors.

Cataloging-in-Publication Data on file with the Library of Congress.

Contents

Preface ix

1

Functions 1

- 1.1 Functions and Their Graphs 1
- 1.2 Combining Functions; Shifting and Scaling Graphs 14
- 1.3 Trigonometric Functions 21
- 1.4 Graphing with Software 29
- 1.5 Exponential Functions 33
- 1.6 Inverse Functions and Logarithms 38

Also available: A.1 Real Numbers and the Real Line, A.3 Lines and Circles

2

Limits and Continuity 51

- 2.1 Rates of Change and Tangent Lines to Curves 51
- 2.2 Limit of a Function and Limit Laws 58
- 2.3 The Precise Definition of a Limit 69
- 2.4 One-Sided Limits 78
- 2.5 Continuity 85
- 2.6 Limits Involving Infinity; Asymptotes of Graphs 97

Questions to Guide Your Review 110

Practice Exercises 111

Additional and Advanced Exercises 113

Also available: A.5 Proofs of Limit Theorems

3

Derivatives 116

- 3.1 Tangent Lines and the Derivative at a Point 116
- 3.2 The Derivative as a Function 120
- 3.3 Differentiation Rules 129
- 3.4 The Derivative as a Rate of Change 139
- 3.5 Derivatives of Trigonometric Functions 148
- 3.6 The Chain Rule 154
- 3.7 Implicit Differentiation 162
- 3.8 Derivatives of Inverse Functions and Logarithms 167
- 3.9 Inverse Trigonometric Functions 177
- 3.10 Related Rates 184
- 3.11 Linearization and Differentials 192

Questions to Guide Your Review 203

Practice Exercises 204

Additional and Advanced Exercises 208

4

Applications of Derivatives 212

- 4.1 Extreme Values of Functions on Closed Intervals 212
- 4.2 The Mean Value Theorem 220
- 4.3 Monotonic Functions and the First Derivative Test 228
- 4.4 Concavity and Curve Sketching 233
- 4.5 Indeterminate Forms and L'Hôpital's Rule 246
- 4.6 Applied Optimization 255
- 4.7 Newton's Method 266
- 4.8 Antiderivatives 271
- Questions to Guide Your Review* 281
- Practice Exercises* 282
- Additional and Advanced Exercises* 286

5

Integrals 290

- 5.1 Area and Estimating with Finite Sums 290
- 5.2 Sigma Notation and Limits of Finite Sums 300
- 5.3 The Definite Integral 307
- 5.4 The Fundamental Theorem of Calculus 320
- 5.5 Indefinite Integrals and the Substitution Method 332
- 5.6 Definite Integral Substitutions and the Area Between Curves 339
- Questions to Guide Your Review* 349
- Practice Exercises* 350
- Additional and Advanced Exercises* 353

6

Applications of Definite Integrals 356

- 6.1 Volumes Using Cross-Sections 356
- 6.2 Volumes Using Cylindrical Shells 367
- 6.3 Arc Length 375
- 6.4 Areas of Surfaces of Revolution 381
- 6.5 Work 386
- 6.6 Moments and Centers of Mass 392
- Questions to Guide Your Review* 401
- Practice Exercises* 402
- Additional and Advanced Exercises* 403

7

Integrals and Transcendental Functions 405

- 7.1 The Logarithm Defined as an Integral 405
- 7.2 Exponential Change and Separable Differential Equations 415
- 7.3 Hyperbolic Functions 425
- Questions to Guide Your Review* 433
- Practice Exercises* 433
- Additional and Advanced Exercises* 434
- Also available: B.1 Relative Rates of Growth*

8

Techniques of Integration 436

- 8.1 Integration by Parts 437
- 8.2 Trigonometric Integrals 445
- 8.3 Trigonometric Substitutions 451
- 8.4 Integration of Rational Functions by Partial Fractions 456
- 8.5 Integral Tables and Computer Algebra Systems 463
- 8.6 Numerical Integration 469
- 8.7 Improper Integrals 478

Questions to Guide Your Review 489

Practice Exercises 490

Additional and Advanced Exercises 492

Also available: B.2 Probability

9

Infinite Sequences and Series 495

- 9.1 Sequences 495
- 9.2 Infinite Series 508
- 9.3 The Integral Test 518
- 9.4 Comparison Tests 524
- 9.5 Absolute Convergence; The Ratio and Root Tests 529
- 9.6 Alternating Series and Conditional Convergence 536
- 9.7 Power Series 543
- 9.8 Taylor and Maclaurin Series 554
- 9.9 Convergence of Taylor Series 559
- 9.10 Applications of Taylor Series 565

Questions to Guide Your Review 574

Practice Exercises 575

Additional and Advanced Exercises 577

Also available: A.6 Commonly Occurring Limits

10

Parametric Equations and Polar Coordinates 580

- 10.1 Parametrizations of Plane Curves 580
- 10.2 Calculus with Parametric Curves 588
- 10.3 Polar Coordinates 598
- 10.4 Graphing Polar Coordinate Equations 602
- 10.5 Areas and Lengths in Polar Coordinates 606

Questions to Guide Your Review 611

Practice Exercises 611

Additional and Advanced Exercises 613

Also available: A.4 Conic Sections, B.3 Conics in Polar Coordinates

11 Vectors and the Geometry of Space 614

- 11.1 Three-Dimensional Coordinate Systems 614
 - 11.2 Vectors 619
 - 11.3 The Dot Product 628
 - 11.4 The Cross Product 636
 - 11.5 Lines and Planes in Space 642
 - 11.6 Cylinders and Quadric Surfaces 651
 - Questions to Guide Your Review* 657
 - Practice Exercises* 657
 - Additional and Advanced Exercises* 659
- Also available: A.9 The Distributive Law for Vector Cross Products*

12 Vector-Valued Functions and Motion in Space 662

- 12.1 Curves in Space and Their Tangents 662
- 12.2 Integrals of Vector Functions; Projectile Motion 671
- 12.3 Arc Length in Space 678
- 12.4 Curvature and Normal Vectors of a Curve 682
- 12.5 Tangential and Normal Components of Acceleration 687
- 12.6 Velocity and Acceleration in Polar Coordinates 690
- Questions to Guide Your Review* 694
- Practice Exercises* 694
- Additional and Advanced Exercises* 696

13 Partial Derivatives 697

- 13.1 Functions of Several Variables 697
 - 13.2 Limits and Continuity in Higher Dimensions 705
 - 13.3 Partial Derivatives 714
 - 13.4 The Chain Rule 726
 - 13.5 Directional Derivatives and Gradient Vectors 736
 - 13.6 Tangent Planes and Differentials 744
 - 13.7 Extreme Values and Saddle Points 754
 - 13.8 Lagrange Multipliers 763
 - Questions to Guide Your Review* 773
 - Practice Exercises* 773
 - Additional and Advanced Exercises* 777
- Also available: A.10 The Mixed Derivative Theorem and the Increment Theorem, B.4 Taylor's Formula for Two Variables, B.5 Partial Derivatives with Constrained Variables*

14 Multiple Integrals 779

- 14.1 Double and Iterated Integrals over Rectangles 779
- 14.2 Double Integrals over General Regions 784
- 14.3 Area by Double Integration 793
- 14.4 Double Integrals in Polar Form 796
- 14.5 Triple Integrals in Rectangular Coordinates 803
- 14.6 Applications 813
- 14.7 Triple Integrals in Cylindrical and Spherical Coordinates 820
- 14.8 Substitutions in Multiple Integrals 832
 - Questions to Guide Your Review* 841
 - Practice Exercises* 842
 - Additional and Advanced Exercises* 844

15 Integrals and Vector Fields 847

- 15.1 Line Integrals of Scalar Functions 847
- 15.2 Vector Fields and Line Integrals: Work, Circulation, and Flux 854
- 15.3 Path Independence, Conservative Fields, and Potential Functions 867
- 15.4 Green's Theorem in the Plane 878
- 15.5 Surfaces and Area 890
- 15.6 Surface Integrals 900
- 15.7 Stokes' Theorem 910
- 15.8 The Divergence Theorem and a Unified Theory 923
 - Questions to Guide Your Review* 934
 - Practice Exercises* 934
 - Additional and Advanced Exercises* 937

16 First-Order Differential Equations 16-1 (Online at bit.ly/2pzY1Eq)

- 16.1 Solutions, Slope Fields, and Euler's Method 16-1
- 16.2 First-Order Linear Equations 16-9
- 16.3 Applications 16-15
- 16.4 Graphical Solutions of Autonomous Equations 16-21
- 16.5 Systems of Equations and Phase Planes 16-28
 - Questions to Guide Your Review* 16-34
 - Practice Exercises* 16-34
 - Additional and Advanced Exercises* 16-36

17 Second-Order Differential Equations 17-1 (Online at bit.ly/2IHCJyE)

- 17.1 Second-Order Linear Equations 17-1
- 17.2 Nonhomogeneous Linear Equations 17-7
- 17.3 Applications 17-15
- 17.4 Euler Equations 17-22
- 17.5 Power-Series Solutions 17-24

Appendix A AP-1

- A.1** Real Numbers and the Real Line AP-1
- A.2** Mathematical Induction AP-6
- A.3** Lines and Circles AP-9
- A.4** Conic Sections AP-16
- A.5** Proofs of Limit Theorems AP-23
- A.6** Commonly Occurring Limits AP-26
- A.7** Theory of the Real Numbers AP-27
- A.8** Complex Numbers AP-30
- A.9** The Distributive Law for Vector Cross Products AP-38
- A.10** The Mixed Derivative Theorem and the Increment Theorem AP-39

Appendix B B-1 (Online at bit.ly/2IDD18w)

- B.1** Relative Rates of Growth B-1
- B.2** Probability B-6
- B.3** Conics in Polar Coordinates B-19
- B.4** Taylor's Formula for Two Variables B-25
- B.5** Partial Derivatives with Constrained Variables B-29

Answers to Odd-Numbered Exercises AN-1**Applications Index** AI-1**Subject Index** I-1**Credits** C-1**A Brief Table of Integrals** T-1

Preface

University Calculus: Early Transcendentals, Fourth Edition, provides a streamlined treatment of the material in a standard three-semester or four-quarter STEM-oriented course. As the title suggests, the book aims to go beyond what many students may have seen at the high school level. The book emphasizes mathematical precision and conceptual understanding, supporting these goals with clear explanations and examples and carefully crafted exercise sets.

Generalization drives the development of calculus and of mathematical maturity and is pervasive in this text. Slopes of lines generalize to slopes of curves, lengths of line segments to lengths of curves, areas and volumes of regular geometric figures to areas and volumes of shapes with curved boundaries, and finite sums to series. Plane analytic geometry generalizes to the geometry of space, and single variable calculus to the calculus of many variables. Generalization weaves together the many threads of calculus into an elegant tapestry that is rich in ideas and their applications.

Mastering this beautiful subject is its own reward, but the real gift of studying calculus is acquiring the ability to think logically and precisely; understanding what is defined, what is assumed, and what is deduced; and learning how to generalize conceptually. We intend this text to encourage and support those goals.

New to This Edition

We welcome to this edition two new co-authors: Christopher Heil from Georgia Institute of Technology and Przemyslaw Bogacki from Old Dominion University. Heil's focus was primarily on the development of the text itself, while Bogacki focused on the MyLab™ Math course.

Christopher Heil has been involved in teaching calculus, linear algebra, analysis, and abstract algebra at Georgia Tech since 1993. He is an experienced author and served as a consultant on the previous edition of this text. His research is in harmonic analysis, including time-frequency analysis, wavelets, and operator theory.

Przemyslaw Bogacki joined the faculty at Old Dominion University in 1990. He has taught calculus, linear algebra, and numerical methods. He is actively involved in applications of technology in collegiate mathematics. His areas of research include computer-aided geometric design and numerical solution of initial value problems for ordinary differential equations.

This is a substantial revision. Every word, symbol, and figure was revisited to ensure clarity, consistency, and conciseness. Additionally, we made the following text-wide changes:

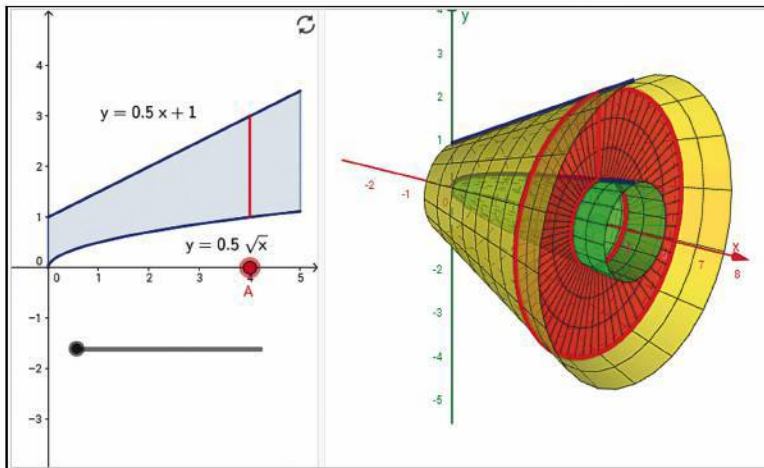
- Updated graphics to bring out clear visualization and mathematical correctness.
- Added new types of homework exercises throughout, including many that are geometric in nature. The new exercises are not just more of the same, but rather give different perspectives and approaches to each topic. In preparing this edition, we analyzed aggregated student usage and performance data from MyLab Math for the previous edition of the text. The results of this analysis increased both the quality and the quantity of the exercises.

- Added short URLs to historical links, thus enabling students to navigate directly to on-line information.
- Added new annotations in blue type throughout the text to guide the reader through the process of problem solution and emphasize that each step in a mathematical argument is rigorously justified.

New To MyLab Math

Many improvements have been made to the overall functionality of MyLab Math since the previous edition. We have also enhanced and improved the content specific to this text.

- Every online exercise in the course was reviewed for accuracy and alignment with the text by author Przemyslaw Bogacki.
- Instructors now have more exercises than ever to choose from in assigning homework. There are approximately 8550 assignable exercises in MyLab Math, 490 of which are new to this edition.
- The MyLab Math exercise-scoring engine has been updated to allow for more robust coverage of certain topics, including differential equations.



- A full suite of Interactive Figures have been added to support teaching and learning. The figures are designed to be used in lecture as well as by students independently. The figures are editable via the freely available GeoGebra software.
- Enhanced Sample Assignments include just-in-time prerequisite review, help keep skills fresh with spaced practice of key concepts, and provide opportunities to work exercises without learning aids (to help students develop confidence in their ability to solve problems independently).
- Additional Conceptual Questions augment the text exercises to focus on deeper, theoretical understanding of the key concepts in calculus. These questions

were written by faculty at Cornell University under an NSF grant. They are also assignable through Learning Catalytics.

- This MyLab Math course contains pre-made quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified.
- Additional Setup & Solve exercises now appear in many sections. These exercises require students to show how they set up a problem, as well as the solution itself, better mirroring what is required of students on tests.
- PowerPoint lecture slides have been expanded to include examples as well as key theorems, definitions, and figures.
- Over 200 new instructional videos augment the already robust collection within the course. These videos support the overall approach of the text—specifically, they go beyond routine procedures to show students how to generalize and connect key concepts.

Content Enhancements

Chapter 1

- Shortened 1.4 to focus on issues arising in use of mathematical software, and potential pitfalls. Removed peripheral material on regression, along with associated exercises.
- Clarified explanation of definition of exponential function in 1.5.
- Replaced \sin^{-1} notation for the inverse sine function with \arcsin as default notation in 1.6, and similarly for other trig functions.

Chapter 2

- Added definition of average speed in 2.1.
- Updated definition of limits to allow for arbitrary domains. The definition of limits is now consistent with the definition in multivariable domains later in the text and with more general mathematical usage.
- Reworded limit and continuity definitions to remove implication symbols and improve comprehension.
- Replaced Example 1 in 2.4, reordered, and added new Example 2 to clarify one-sided limits.
- Added new Example 7 in 2.4 to illustrate limits of ratios of trig functions.
- Rewrote Example 11 in 2.5 to solve the equation by finding a zero, consistent with the previous discussion.

Chapter 3

- Clarified relation of slope and rate of change.
- Added new Figure 3.9 using the square root function to illustrate vertical tangent lines.
- Added figure of $x \sin(1/x)$ in 3.2 to illustrate how oscillation can lead to non-existence of a derivative of a continuous function.
- Revised product rule to make order of factors consistent throughout text, including later dot product and cross product formulas.
- Expanded Example 7 in 3.8 to clarify the computation of the derivative of x^x .
- Updated and improved related rates problem strategies in 3.10, and correspondingly revised Examples 2–6.

Chapters 4 & 5

- Added summary to 4.1.
- Added new Example 3 with new Figure 4.27, and Example 12 with new Figure 4.35, to give basic and advanced examples of concavity.
- Updated and improved strategies for solving applied optimization problems in 4.6.
- Improved discussion in 5.4 and added new Figure 5.18 to illustrate the Mean Value Theorem.

Chapters 6 & 7

- Clarified cylindrical shell method.
- Converted Example 4 in 6.5 to metric units.
- Added introductory discussion of mass distribution along a line, with figure, in 6.6.
- Clarified discussion of separable differential equations in 7.2.

Chapter 8

- Updated Integration by Parts discussion in 8.2 to emphasize $u(x)v'(x) dx$ form rather than $u dv$. Rewrote Examples 1–3 accordingly.
- Removed discussion of tabular integration, along with associated exercises.
- Updated discussion in 8.4 on how to find constants in the method of partial fractions, and clarified the corresponding calculations in Example 1.

Chapter 9

- Clarified the different meanings of sequence and series.
- Added new Figure 9.9 to illustrate sum of a series as area of a histogram.
- Added to 9.3 a discussion on the importance of bounding errors in approximations.
- Added new Figure 9.13 illustrating how to use integrals to bound remainder terms of partial sums.
- Rewrote Theorem 10 in 9.4 to bring out similarity to the integral comparison test.

- Added new Figure 9.16 to illustrate the differing behaviors of the harmonic and alternating harmonic series.
- Renamed the n th-Term Test the “ n th-Term Test for Divergence” to emphasize that it says nothing about convergence.
- Added new Figure 9.19 to illustrate polynomials converging to $\ln(1 + x)$, which illustrates convergence on the half-open interval $(-1, 1]$.
- Used red dots and intervals to indicate intervals and points where divergence occurs and blue to indicate convergence throughout Chapter 9.
- Added new Figure 9.21 to show the six different possibilities for an interval of convergence.
- Changed the name of 9.10 to “Applications of Taylor Series.”

Chapter 10

- Added new Example 1 and Figure 10.2 in 10.1 to give a straightforward first Example of a parametrized curve.
- Updated area formulas for polar coordinates to include conditions for positive r and non-overlapping θ .
- Added new Example 3 and Figure 10.37 in 10.4 to illustrate intersections of polar curves.
- Moved Section 10.6 (“Conics in Polar Coordinates”), which our data showed is seldom used, to online Appendix B (bit.ly/2IDD18w).

Chapters 11 & 12

- Added new Figure 11.13b to show the effect of scaling a vector.
- Added new Example 7 and Figure 11.26 in 11.3 to illustrate projection of a vector.
- Added discussion on general quadric surfaces in 11.6, with new Example 4 and new Figure 11.48 illustrating the description of an ellipsoid not centered at the origin via completing the square.
- Added sidebars on how to pronounce Greek letters such as kappa and tau.

Chapter 13

- Elaborated on discussion of open and closed regions in 13.1.
- Added a Composition Rule to Theorem 1 and expanded Example 1 in 13.2.
- Expanded Example 8 in 13.3.
- Clarified Example 6 in 13.7.
- Standardized notation for evaluating partial derivatives, gradients, and directional derivatives at a point, throughout the chapter.
- Renamed “branch diagrams” as “dependency diagrams” to clarify that they capture dependence of variables.

Chapter 14

- Added new Figure 14.21b to illustrate setting up limits of a double integral.
- In 14.5, added new Example 1, modified Examples 2 and 3, and added new Figures 14.31, 14.32, and 14.33 to give basic examples of setting up limits of integration for a triple integral.

Chapter 15

- Added new Figure 15.4 to illustrate a line integral of a function, new Figure 15.17 to illustrate a gradient field, and new Figure 15.18 to illustrate a line integral of a vector field.
- Clarified notation for line integrals in 15.2.
- Added discussion of the sign of potential energy in 15.3.
- Rewrote solution of Example 3 in 15.4 to clarify its connection to Green’s Theorem.
- Updated discussion of surface orientation in 15.6, along with Figure 15.52.

Appendices

- Rewrote Appendix A.8 on complex numbers.
- Added online Appendix B (bit.ly/2IDD18w) containing additional topics. These topics are supported fully in MyLab Math.

Continuing Features

Rigor The level of rigor is consistent with that of earlier editions. We continue to distinguish between formal and informal discussions and to point out their differences. We think starting with a more intuitive, less formal approach helps students understand a new or difficult concept so they can then appreciate its full mathematical precision and outcomes. We pay attention to defining ideas carefully and to proving theorems appropriate

for calculus students, while mentioning deeper or subtler issues they would study in a more advanced course. Our organization and distinctions between informal and formal discussions give the instructor a degree of flexibility in the amount and depth of coverage of the various topics. For example, although we do not prove the Intermediate Value Theorem or the Extreme Value Theorem for continuous functions on a closed finite interval, we do state these theorems precisely, illustrate their meanings in numerous examples, and use them to prove other important results. Furthermore, for those instructors who desire greater depth of coverage, in Appendix A.7 we discuss the reliance of these theorems on the completeness of the real numbers.

Writing Exercises Writing exercises placed throughout the text ask students to explore and explain a variety of calculus concepts and applications. In addition, the end of each chapter includes a list of questions that invite students to review and summarize what they have learned. Many of these exercises make good writing assignments.

End-Of-Chapter Reviews In addition to problems appearing after each section, each chapter culminates with review questions, practice exercises covering the entire chapter, and a series of Additional and Advanced Exercises with more challenging or synthesizing problems.

Writing And Applications This text continues to be easy to read, conversational, and mathematically rich. Each new topic is motivated by clear, easy-to-understand examples and is then reinforced by its application to real-world problems of immediate interest to students. A hallmark of this text is the application of calculus to science and engineering. These applied problems have been updated, improved, and extended continually over the last several editions.

Technology In a course using this text, technology can be incorporated according to the taste of the instructor. Each section contains exercises requiring the use of technology; these are marked with a **T** if suitable for calculator or computer use, or they are labeled **Computer Explorations** if a computer algebra system (CAS, such as *Maple* or *Mathematica*) is required.

Acknowledgments

We are grateful to Duane Kouba, who created many of the new exercises. We would also like to express our thanks to the people who made many valuable contributions to this edition as it developed through its various stages:

Accuracy Checkers

Jennifer Blue
Thomas Wegleitner

Reviewers for the Fourth Edition

Scott Allen, *Chattahoochee Technical College*
Alessandro Arsie, *University of Toledo*
Doug Baldwin, *SUNY Geneseo*
Imad Benjelloun, *Delaware Valley University*
Robert J. Brown, Jr., *East Georgia State University*
Jason Froman, *Lamesa High School*
Morag Fulton, *Ivy Tech Community College*
Michael S. Eusebio, *Ivy Tech Community College*

Laura Hauser, *University of Tampa*
Steven Heilman, *UCLA*
Sandeep Holay, *Southeast Community College*
David Horntrop, *New Jersey Institute of Technology*
Eric Hutchinson, *College of Southern Nevada*
Michael A. Johnston, *Pensacola State College*
Eric B. Kahn, *Bloomsburg University*
Colleen Kirk, *California Polytechnic University*
Weidong Li, *Old Dominion University*
Mark McConnell, *Princeton University*
Tamara Miller, *Ivy Tech Community College - Columbus*
Neils Martin Møller, *Princeton University*
James G. O'Brien, *Wentworth Institute of Technology*
Nicole M. Panza, *Francis Marion University*
Steven Riley, *Chattahoochee Technical College*
Alan Saleski, *Loyola University of Chicago*
Claus Schubert, *SUNY Cortland*
Ruth Trubnik, *Delaware Valley University*
Alan Von Hermann, *Santa Clara University*
Don Gayan Wilathgamuwa, *Montana State University*
James Wilson, *Iowa State University*

Dedication

We regret that prior to the writing of this edition, our co-author Maurice Weir passed away. Maury was dedicated to achieving the highest possible standards in the presentation of mathematics. He insisted on clarity, rigor, and readability. Maury was a role model to his students, his colleagues, and his co-authors. He was very proud of his daughters, Maia Coyle and Renee Waina, and of his grandsons, Matthew Ryan and Andrew Dean Waina. He will be greatly missed.



MyLab Math Online

Course for *University Calculus: Early Transcendentals, 4e*

(access code required)

MyLab™ Math is the teaching and learning platform that empowers instructors to reach *every* student. By combining trusted author content with digital tools and a flexible platform, MyLab Math for *University Calculus: Early Transcendentals, 4e* personalizes the learning experience and improves results for each student.

PREPAREDNESS

One of the biggest challenges in calculus courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their course work. MyLab Math supports students with just-in-time remediation and key-concept review.

Integrated Review

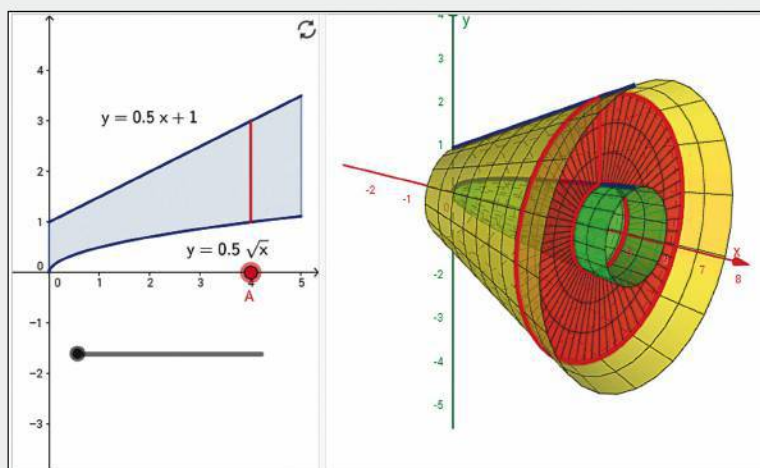
This MyLab course features pre-made, assignable (and editable) quizzes to assess the prerequisite skills needed for each chapter, plus personalized remediation for any gaps in skills that are identified. Each student, therefore, receives just the help that he or she needs—no more, no less.

DEVELOPING DEEPER UNDERSTANDING

MyLab Math provides content and tools that help students build a deeper understanding of course content than would otherwise be possible.

NEW! Interactive Figures

A full suite of Interactive Figures was added to illustrate key concepts and allow manipulation. Designed in the freely available GeoGebra software, these figures can be used in lecture as well as by students independently. Videos that use the Interactive Figures to explain key concepts are also included. The figures were created by Marc Renault (Shippensburg University), Steve Phelps (University of Cincinnati), Kevin Hopkins (Southwest Baptist University), and Tim Brzezinski (Berlin High School, CT).



Exercises with Immediate Feedback

Homework and practice exercises for this text regenerate algorithmically to give students unlimited opportunity for practice and mastery. MyLab Math provides helpful feedback when students enter incorrect answers, and it includes the optional learning aids Help Me Solve This, View an Example, videos, and the eBook.

UPDATED! Assignable Exercises

All online exercises were reviewed for accuracy and fidelity to the text by author Przemyslaw Bogacki.

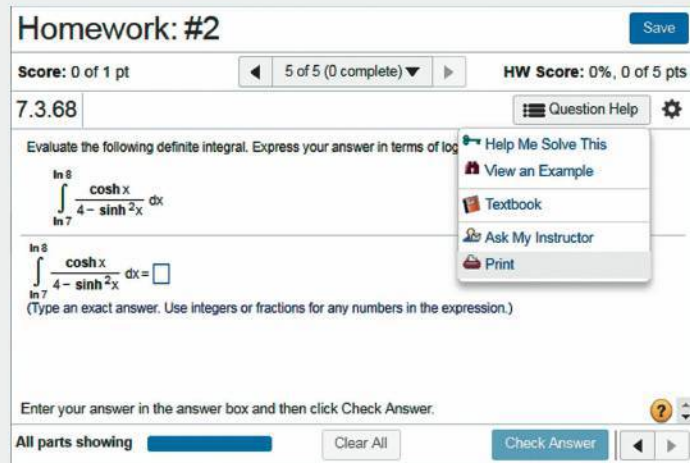
Additionally, the authors analyzed aggregated student usage and performance data from MyLab Math for the previous edition of this text. The results of this analysis helped increase the quality and quantity of the text and of the MyLab exercises and learning aids that matter most to instructors and students.

NEW! Enhanced Sample Assignments

These section-level assignments include just-in-time prerequisite review, help keep skills fresh with spaced practice of key concepts, and provide opportunities to work exercises without learning aids so students check their understanding. They are assignable and editable within MyLab Math.

ENHANCED! Setup & Solve Exercises

These exercises require students to show how they set up a problem, as well as the solution itself, better mirroring what is required on tests.



Homework: #2 Save

Score: 0 of 1 pt 5 of 5 (0 complete) HW Score: 0%, 0 of 5 pts

7.3.68 Question Help

Evaluate the following definite integral. Express your answer in terms of log.

$$\int_{\ln 7}^{\ln 8} \frac{\cosh x}{4 - \sinh^2 x} dx$$

$\int_{\ln 7}^{\ln 8} \frac{\cosh x}{4 - \sinh^2 x} dx = \square$

(Type an exact answer. Use integers or fractions for any numbers in the expression.)

Enter your answer in the answer box and then click Check Answer.

All parts showing Clear All Check Answer

Find the volume of the following solid.

The solid bounded by the paraboloid $z = 27 - 3x^2 - 3y^2$ and the plane $z = 24$

Set up the double integral, in polar coordinates, that is used to find the volume.

$$\int_0^{2\pi} \int_0^1 (3r - 3r^3) dr d\theta$$

(Type exact answers.)

$$V = \frac{3}{2}\pi \text{ units}^3$$

(Type an exact answer.)



NEW! Additional Conceptual Questions

Additional Conceptual Questions focus on deeper, theoretical understanding of the key concepts in calculus. These questions were written by faculty at Cornell University under an NSF grant and are also assignable through Learning Catalytics™.

UPDATED! Instructional Videos

Hundreds of videos are available as learning aids within exercises and for self-study. The Guide to Video-Based Assignments makes it easy to assign videos for homework in MyLab Math by showing which MyLab exercises correspond to each video.

UPDATED! Technology Manuals (downloadable)

- Maple™ Manual and Projects by Kevin Reeves, East Texas Baptist University
- Mathematica® Manual and Projects by Todd Lee, Elon University
- TI-Graphing Calculator Manual by Elaine McDonald-Newman, Sonoma State University

These manuals cover *Maple 2017*, *Mathematica 11*, and the TI-84 Plus and TI-89, respectively. Each manual provides detailed guidance for integrating the software package or graphing calculator throughout the course, including syntax and commands. The projects include instructions and ready-made application files for Maple and Mathematica. The files can be downloaded from within MyLab Math.

Student's Solutions Manuals (softcover and downloadable)

Single Variable Calculus: Early Transcendentals (Chapters 1–10)

ISBN: 0-13-516613-6 | 978-0-13-516613-0

Multivariable Calculus (Chapters 9–15) ISBN: 0-13-516663-2 | 978-0-13-516663-5

The Student's Solutions Manuals contain worked-out solutions to all the odd-numbered exercises. These manuals are available in print and can be downloaded from within MyLab Math.

SUPPORTING INSTRUCTION

MyLab Math comes from an experienced partner with educational expertise and an eye on the future. It provides resources to help you assess and improve student results at every turn and unparalleled flexibility to create a course tailored to you and your students.

UPDATED! PowerPoint Lecture Slides (downloadable)

Classroom presentation slides feature key concepts, examples, definitions, figures, and tables from this text. They can be downloaded from within MyLab Math or from Pearson's online catalog, www.pearson.com.



Learning Catalytics

Now included in all MyLab Math courses, this student response tool uses students' smartphones, tablets, or laptops to engage them in more interactive tasks and thinking during lecture. Learning Catalytics™ fosters student engagement and peer-to-peer learning with real-time analytics.

Comprehensive Gradebook

The gradebook includes enhanced reporting functionality, such as item analysis and a reporting dashboard, to allow you to efficiently manage your course. Student performance data is presented at the class, section, and program levels in an accessible, visual manner so you'll have the information you need to keep your students on track.

TestGen

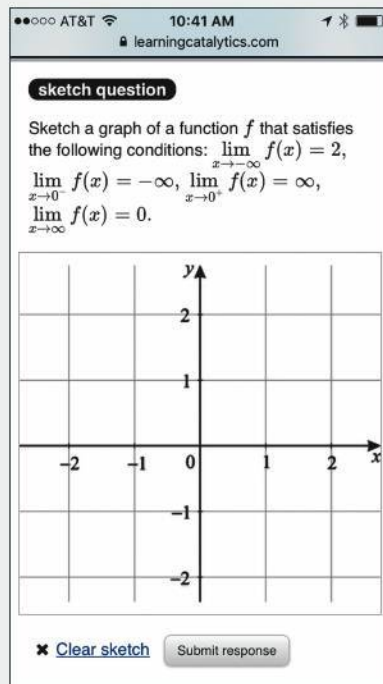
TestGen® (www.pearson.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, enabling instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson's online catalog, www.pearson.com. The questions are also assignable in MyLab Math.

Instructor's Solutions Manual (downloadable)

The Instructor's Solutions Manual contains complete solutions to the exercises in Chapters 1–17. It can be downloaded from within MyLab Math or from Pearson's online catalog, www.pearson.com.

Accessibility

Pearson works continuously to ensure our products are as accessible as possible to all students. We are working toward achieving WCAG 2.0 Level AA and Section 508 standards, as expressed in the Pearson Guidelines for Accessible Educational Web Media, www.pearson.com/mylab/math/accessibility.



1

Functions

OVERVIEW In this chapter we review what functions are and how they are visualized as graphs, how they are combined and transformed, and ways they can be classified.

1.1 Functions and Their Graphs

Functions are a tool for describing the real world in mathematical terms. A function can be represented by an equation, a graph, a numerical table, or a verbal description; we will use all four representations throughout this text. This section reviews these ideas.

Functions; Domain and Range

The temperature at which water boils depends on the elevation above sea level. The interest paid on a cash investment depends on the length of time the investment is held. The area of a circle depends on the radius of the circle. The distance an object travels depends on the elapsed time.

In each case, the value of one variable quantity, say y , depends on the value of another variable quantity, which we often call x . We say that “ y is a function of x ” and write this symbolically as

$$y = f(x) \quad (\text{“}y \text{ equals } f \text{ of } x\text{”}).$$

The symbol f represents the function, the letter x is the **independent variable** representing the input value to f , and y is the **dependent variable** or output value of f at x .

DEFINITION A **function** f from a set D to a set Y is a rule that assigns a *unique* value $f(x)$ in Y to each x in D .

The set D of all possible input values is called the **domain** of the function. The set of all output values of $f(x)$ as x varies throughout D is called the **range** of the function. The range might not include every element in the set Y . The domain and range of a function can be any sets of objects, but often in calculus they are sets of real numbers interpreted as points of a coordinate line. (In Chapters 12–15, we will encounter functions for which the elements of the sets are points in the plane, or in space.)

Often a function is given by a formula that describes how to calculate the output value from the input variable. For instance, the equation $A = \pi r^2$ is a rule that calculates the area A of a circle from its radius r . When we define a function $y = f(x)$ with a formula and the domain is not stated explicitly or restricted by context, the domain is assumed to be

the largest set of real x -values for which the formula gives real y -values. This is called the **natural domain** of f . If we want to restrict the domain in some way, we must say so. The domain of $y = x^2$ is the entire set of real numbers. To restrict the domain of the function to, say, positive values of x , we would write “ $y = x^2, x > 0$.”

Changing the domain to which we apply a formula usually changes the range as well. The range of $y = x^2$ is $[0, \infty)$. The range of $y = x^2, x \geq 2$, is the set of all numbers obtained by squaring numbers greater than or equal to 2. In set notation (see Appendix A.1), the range is $\{x^2 | x \geq 2\}$ or $\{y | y \geq 4\}$ or $[4, \infty)$.

When the range of a function is a set of real numbers, the function is said to be **real-valued**. The domains and ranges of most real-valued functions we consider are intervals or combinations of intervals. Sometimes the range of a function is not easy to find.

A function f is like a machine that produces an output value $f(x)$ in its range whenever we feed it an input value x from its domain (Figure 1.1). The function keys on a calculator give an example of a function as a machine. For instance, the \sqrt{x} key on a calculator gives an output value (the square root) whenever you enter a nonnegative number x and press the \sqrt{x} key.

A function can also be pictured as an **arrow diagram** (Figure 1.2). Each arrow associates to an element of the domain D a single element in the set Y . In Figure 1.2, the arrows indicate that $f(a)$ is associated with a , $f(x)$ is associated with x , and so on. Notice that a function can have the same *output value* for two different input elements in the domain (as occurs with $f(a)$ in Figure 1.2), but each input element x is assigned a *single* output value $f(x)$.

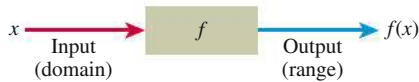


FIGURE 1.1 A diagram showing a function as a kind of machine.

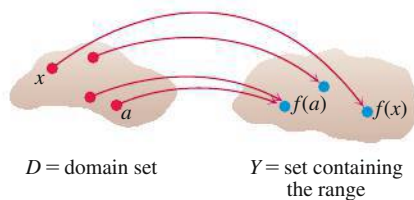


FIGURE 1.2 A function from a set D to a set Y assigns a unique element of Y to each element in D .

EXAMPLE 1 Verify the natural domains and associated ranges of some simple functions. The domains in each case are the values of x for which the formula makes sense.

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Solution The formula $y = x^2$ gives a real y -value for any real number x , so the domain is $(-\infty, \infty)$. The range of $y = x^2$ is $[0, \infty)$ because the square of any real number is nonnegative and every nonnegative number y is the square of its own square root: $y = (\sqrt{y})^2$ for $y \geq 0$.

The formula $y = 1/x$ gives a real y -value for every x except $x = 0$. For consistency in the rules of arithmetic, *we cannot divide any number by zero*. The range of $y = 1/x$, the set of reciprocals of all nonzero real numbers, is the set of all nonzero real numbers, since $y = 1/(1/y)$. That is, for $y \neq 0$ the number $x = 1/y$ is the input that is assigned to the output value y .

The formula $y = \sqrt{x}$ gives a real y -value only if $x \geq 0$. The range of $y = \sqrt{x}$ is $[0, \infty)$ because every nonnegative number is some number's square root (namely, it is the square root of its own square).

In $y = \sqrt{4 - x}$, the quantity $4 - x$ cannot be negative. That is, $4 - x \geq 0$, or $x \leq 4$. The formula gives nonnegative real y -values for all $x \leq 4$. The range of $\sqrt{4 - x}$ is $[0, \infty)$, the set of all nonnegative numbers.

The formula $y = \sqrt{1 - x^2}$ gives a real y -value for every x in the closed interval from -1 to 1 . Outside this domain, $1 - x^2$ is negative and its square root is not a real number. The values of $1 - x^2$ vary from 0 to 1 on the given domain, and the square roots of these values do the same. The range of $\sqrt{1 - x^2}$ is $[0, 1]$. ■

Graphs of Functions

If f is a function with domain D , its **graph** consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f . In set notation, the graph is

$$\{(x, f(x)) \mid x \in D\}.$$

The graph of the function $f(x) = x + 2$ is the set of points with coordinates (x, y) for which $y = x + 2$. Its graph is the straight line sketched in Figure 1.3.

The graph of a function f is a useful picture of its behavior. If (x, y) is a point on the graph, then $y = f(x)$ is the height of the graph above (or below) the point x . The height may be positive or negative, depending on the sign of $f(x)$ (Figure 1.4).

x	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4

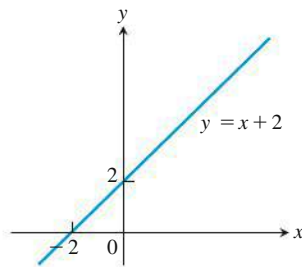


FIGURE 1.3 The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

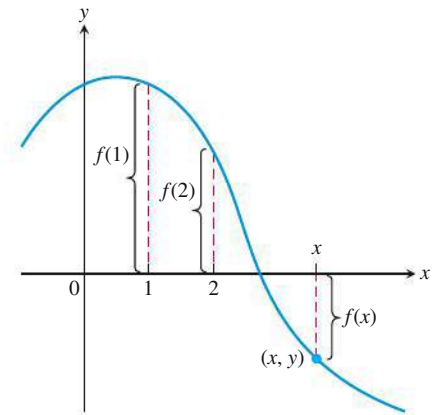


FIGURE 1.4 If (x, y) lies on the graph of f , then the value $y = f(x)$ is the height of the graph above the point x (or below x if $f(x)$ is negative).

EXAMPLE 2 Graph the function $y = x^2$ over the interval $[-2, 2]$.

Solution Make a table of xy -pairs that satisfy the equation $y = x^2$. Plot the points (x, y) whose coordinates appear in the table, and draw a *smooth* curve (labeled with its equation) through the plotted points (see Figure 1.5). ■

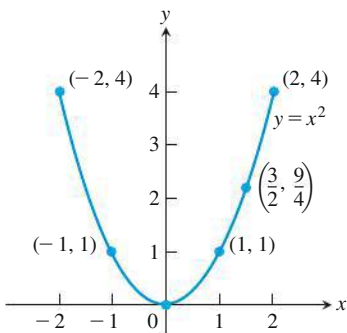
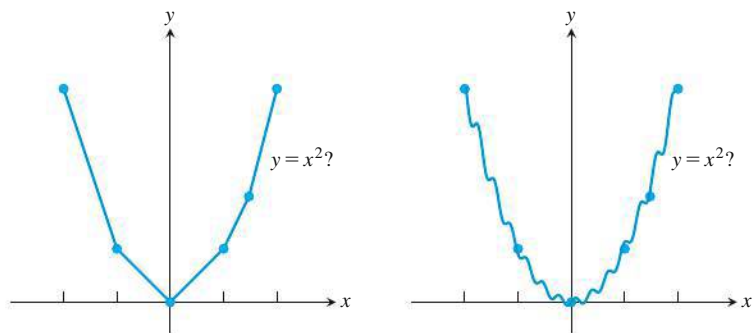


FIGURE 1.5 Graph of the function in Example 2.

How do we know that the graph of $y = x^2$ doesn't look like one of these curves?



To find out, we could plot more points. But how would we then connect *them*? The basic question still remains: How do we know for sure what the graph looks like between the points we plot? Calculus answers this question, as we will see in Chapter 4. Meanwhile, we will have to settle for plotting points and connecting them as best we can.

Time	Pressure
0.00091	-0.080
0.00108	0.200
0.00125	0.480
0.00144	0.693
0.00162	0.816
0.00180	0.844
0.00198	0.771
0.00216	0.603
0.00234	0.368
0.00253	0.099
0.00271	-0.141
0.00289	-0.309
0.00307	-0.348
0.00325	-0.248
0.00344	-0.041
0.00362	0.217
0.00379	0.480
0.00398	0.681
0.00416	0.810
0.00435	0.827
0.00453	0.749
0.00471	0.581
0.00489	0.346
0.00507	0.077
0.00525	-0.164
0.00543	-0.320
0.00562	-0.354
0.00579	-0.248
0.00598	-0.035

Representing a Function Numerically

A function may be represented algebraically by a formula and visually by a graph (Example 2). Another way to represent a function is **numerically**, through a table of values. From an appropriate table of values, a graph of the function can be obtained using the method illustrated in Example 2, possibly with the aid of a computer. The graph consisting of only the points in the table is called a **scatterplot**.

EXAMPLE 3 Musical notes are pressure waves in the air. The data associated with Figure 1.6 give recorded pressure displacement versus time in seconds of a musical note produced by a tuning fork. The table provides a representation of the pressure function (in micropascals) over time. If we first make a scatterplot and then draw a smooth curve that approximates the data points (t, p) from the table, we obtain the graph shown in the figure.

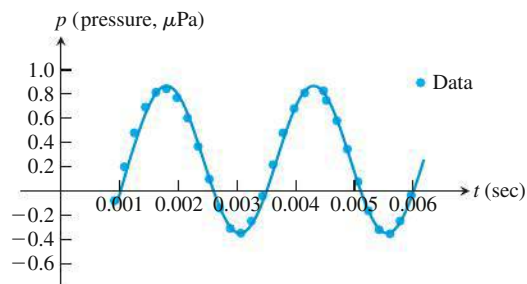


FIGURE 1.6 A smooth curve through the plotted points gives a graph of the pressure function represented by the accompanying table data (Example 3). ■

The Vertical Line Test for a Function

Not every curve in the coordinate plane can be the graph of a function. A function f can have only one value $f(x)$ for each x in its domain, so *no vertical line* can intersect the graph of a function at more than one point. If a is in the domain of the function f , then the vertical line $x = a$ will intersect the graph of f at the single point $(a, f(a))$.

A circle cannot be the graph of a function, since some vertical lines intersect the circle twice. The circle graphed in Figure 1.7a, however, contains the graphs of two functions of x , namely the upper semicircle defined by the function $f(x) = \sqrt{1 - x^2}$ and the lower semicircle defined by the function $g(x) = -\sqrt{1 - x^2}$ (Figures 1.7b and 1.7c).

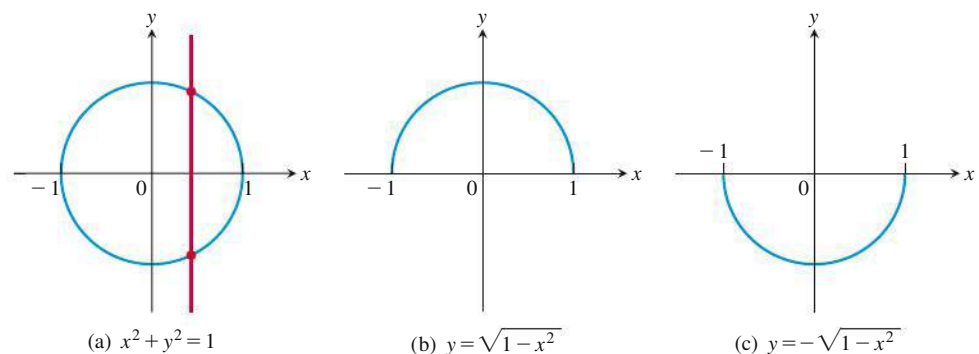


FIGURE 1.7 (a) The circle is not the graph of a function; it fails the vertical line test. (b) The upper semicircle is the graph of the function $f(x) = \sqrt{1 - x^2}$. (c) The lower semicircle is the graph of the function $g(x) = -\sqrt{1 - x^2}$.

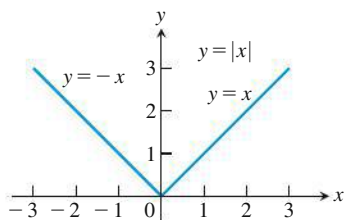


FIGURE 1.8 The absolute value function has domain $(-\infty, \infty)$ and range $[0, \infty)$.

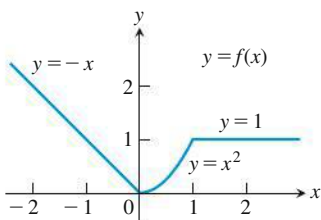


FIGURE 1.9 To graph the function $y = f(x)$ shown here, we apply different formulas to different parts of its domain (Example 4).

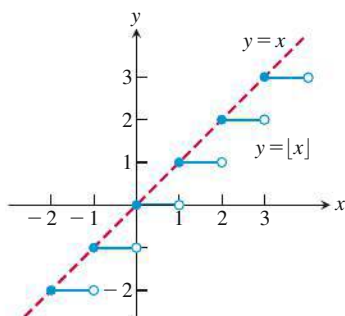


FIGURE 1.10 The graph of the greatest integer function $y = \lfloor x \rfloor$ lies on or below the line $y = x$, so it provides an integer floor for x (Example 5).

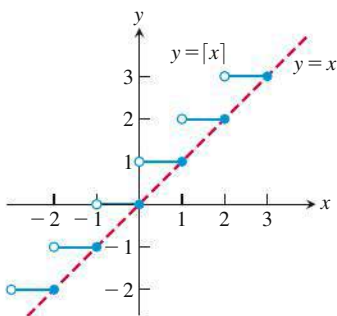


FIGURE 1.11 The graph of the least integer function $y = \lceil x \rceil$ lies on or above the line $y = x$, so it provides an integer ceiling for x (Example 6).

Piecewise-Defined Functions

Sometimes a function is described in pieces by using different formulas on different parts of its domain. One example is the **absolute value function**

$$|x| = \begin{cases} x, & x \geq 0 & \text{First formula} \\ -x, & x < 0 & \text{Second formula} \end{cases}$$

whose graph is given in Figure 1.8. The right-hand side of the equation means that the function equals x if $x \geq 0$, and equals $-x$ if $x < 0$. Piecewise-defined functions often arise when real-world data are modeled. Here are some other examples.

EXAMPLE 4 The function

$$f(x) = \begin{cases} -x, & x < 0 & \text{First formula} \\ x^2, & 0 \leq x \leq 1 & \text{Second formula} \\ 1, & x > 1 & \text{Third formula} \end{cases}$$

is defined on the entire real line but has values given by different formulas, depending on the position of x . The values of f are given by $y = -x$ when $x < 0$, $y = x^2$ when $0 \leq x \leq 1$, and $y = 1$ when $x > 1$. The function, however, is *just one function* whose domain is the entire set of real numbers (Figure 1.9).

EXAMPLE 5 The function whose value at any number x is the *greatest integer less than or equal to* x is called the **greatest integer function** or the **integer floor function**. It is denoted $\lfloor x \rfloor$. Figure 1.10 shows the graph. Observe that

$$\begin{aligned} \lfloor 2.4 \rfloor &= 2, & \lfloor 1.9 \rfloor &= 1, & \lfloor 0 \rfloor &= 0, & \lfloor -1.2 \rfloor &= -2, \\ \lfloor 2 \rfloor &= 2, & \lfloor 0.2 \rfloor &= 0, & \lfloor -0.3 \rfloor &= -1, & \lfloor -2 \rfloor &= -2. \end{aligned}$$

EXAMPLE 6 The function whose value at any number x is the *smallest integer greater than or equal to* x is called the **least integer function** or the **integer ceiling function**. It is denoted $\lceil x \rceil$. Figure 1.11 shows the graph. For positive values of x , this function might represent, for example, the cost of parking x hours in a parking lot that charges \$1 for each hour or part of an hour.

Increasing and Decreasing Functions

If the graph of a function climbs or rises as you move from left to right, we say that the function is *increasing*. If the graph descends or falls as you move from left to right, the function is *decreasing*.

DEFINITIONS Let f be a function defined on an interval I and let x_1 and x_2 be two distinct points in I .

1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be **increasing** on I .
2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is said to be **decreasing** on I .

It is important to realize that the definitions of increasing and decreasing functions must be satisfied for *every* pair of points x_1 and x_2 in I with $x_1 < x_2$. Because we use the inequality $<$ to compare the function values, instead of \leq , it is sometimes said that f is *strictly* increasing or decreasing on I . The interval I may be finite (also called bounded) or infinite (unbounded).

EXAMPLE 7 The function graphed in Figure 1.9 is decreasing on $(-\infty, 0)$ and increasing on $(0, 1)$. The function is neither increasing nor decreasing on the interval $(1, \infty)$ because the function is constant on that interval, and hence the strict inequalities in the definition of increasing or decreasing are not satisfied on $(1, \infty)$. ■

Even Functions and Odd Functions: Symmetry

The graphs of *even* and *odd* functions have special symmetry properties.

DEFINITIONS A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$,

odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.

The names *even* and *odd* come from powers of x . If y is an even power of x , as in $y = x^2$ or $y = x^4$, it is an even function of x because $(-x)^2 = x^2$ and $(-x)^4 = x^4$. If y is an odd power of x , as in $y = x$ or $y = x^3$, it is an odd function of x because $(-x)^1 = -x$ and $(-x)^3 = -x^3$.

The graph of an even function is **symmetric about the y-axis**. Since $f(-x) = f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, y)$ lies on the graph (Figure 1.12a). A reflection across the y-axis leaves the graph unchanged.

The graph of an odd function is **symmetric about the origin**. Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, -y)$ lies on the graph (Figure 1.12b). Equivalently, a graph is symmetric about the origin if a rotation of 180° about the origin leaves the graph unchanged.

Notice that each of these definitions requires that both x and $-x$ be in the domain of f .

EXAMPLE 8 Here are several functions illustrating the definitions.

$$f(x) = x^2$$

Even function: $(-x)^2 = x^2$ for all x ; symmetry about y-axis. So $f(-3) = 9 = f(3)$. Changing the sign of x does not change the value of an even function.

$$f(x) = x^2 + 1$$

Even function: $(-x)^2 + 1 = x^2 + 1$ for all x ; symmetry about y-axis (Figure 1.13a).

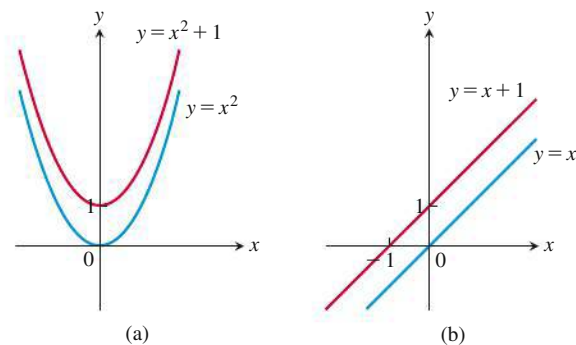


FIGURE 1.13 (a) When we add the constant term 1 to the function $y = x^2$, the resulting function $y = x^2 + 1$ is still even and its graph is still symmetric about the y-axis. (b) When we add the constant term 1 to the function $y = x$, the resulting function $y = x + 1$ is no longer odd, since the symmetry about the origin is lost. The function $y = x + 1$ is also not even (Example 8).

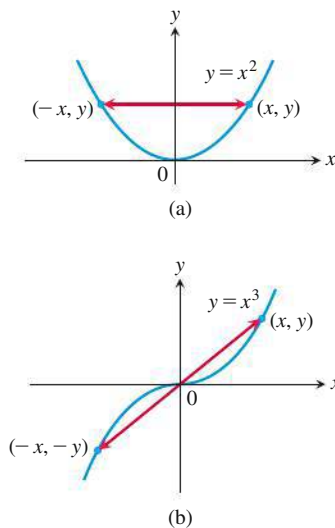


FIGURE 1.12 (a) The graph of $y = x^2$ (an even function) is symmetric about the y-axis. (b) The graph of $y = x^3$ (an odd function) is symmetric about the origin.

- $f(x) = x$ Odd function: $(-x) = -x$ for all x ; symmetry about the origin. So $f(-3) = -3$ while $f(3) = 3$. Changing the sign of x changes the sign of the value of an odd function.
- $f(x) = x + 1$ Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal.
- Not even: $(-x) + 1 \neq x + 1$ for all $x \neq 0$ (Figure 1.13b). ■

Common Functions

A variety of important types of functions are frequently encountered in calculus.

Linear Functions A function of the form $f(x) = mx + b$, where m and b are fixed constants, is called a **linear function**. Figure 1.14a shows an array of lines $f(x) = mx$. Each of these has $b = 0$, so these lines pass through the origin. The function $f(x) = x$, where $m = 1$ and $b = 0$, is called the **identity function**. Constant functions result when the slope is $m = 0$ (Figure 1.14b).

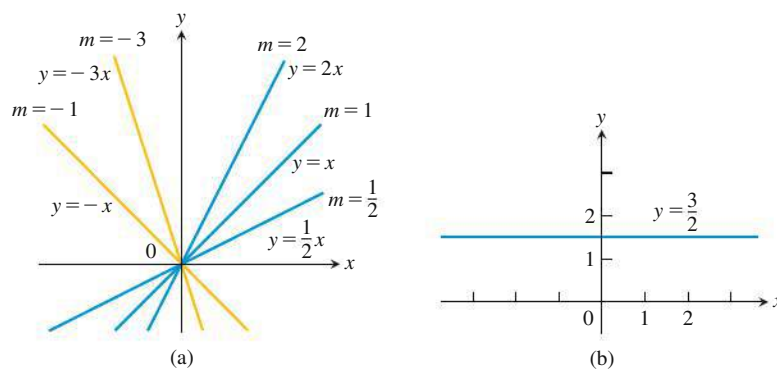


FIGURE 1.14 (a) Lines through the origin with slope m . (b) A constant function with slope $m = 0$.

DEFINITION Two variables y and x are **proportional** (to one another) if one is always a constant multiple of the other—that is, if $y = kx$ for some nonzero constant k .

If the variable y is proportional to the reciprocal $1/x$, then sometimes it is said that y is **inversely proportional** to x (because $1/x$ is the multiplicative inverse of x).

Power Functions A function $f(x) = x^a$, where a is a constant, is called a **power function**. There are several important cases to consider.

- (a) $f(x) = x^a$ with $a = n$, a positive integer.

The graphs of $f(x) = x^n$, for $n = 1, 2, 3, 4, 5$, are displayed in Figure 1.15. These functions are defined for all real values of x . Notice that as the power n gets larger, the curves tend to flatten toward the x -axis on the interval $(-1, 1)$ and to rise more steeply for $|x| > 1$. Each curve passes through the point $(1, 1)$ and through the origin. The graphs of functions with even powers are symmetric about the y -axis; those with odd powers are symmetric about the origin. The even-powered functions are decreasing on the interval $(-\infty, 0]$ and increasing on $[0, \infty)$; the odd-powered functions are increasing over the entire real line $(-\infty, \infty)$.

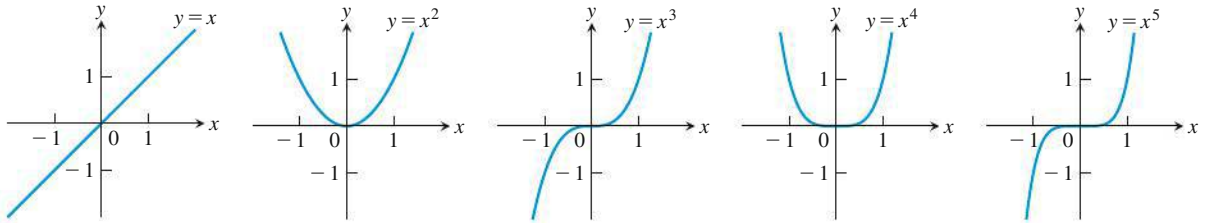


FIGURE 1.15 Graphs of $f(x) = x^n$, $n = 1, 2, 3, 4, 5$, defined for $-\infty < x < \infty$.

(b) $f(x) = x^a$ with $a = -1$ or $a = -2$.

The graphs of the functions $f(x) = x^{-1} = 1/x$ and $f(x) = x^{-2} = 1/x^2$ are shown in Figure 1.16. Both functions are defined for all $x \neq 0$ (you can never divide by zero). The graph of $y = 1/x$ is the hyperbola $xy = 1$, which approaches the coordinate axes far from the origin. The graph of $y = 1/x^2$ also approaches the coordinate axes. The graph of the function $f(x) = 1/x$ is symmetric about the origin; this function is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$. The graph of the function $f(x) = 1/x^2$ is symmetric about the y -axis; this function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$.

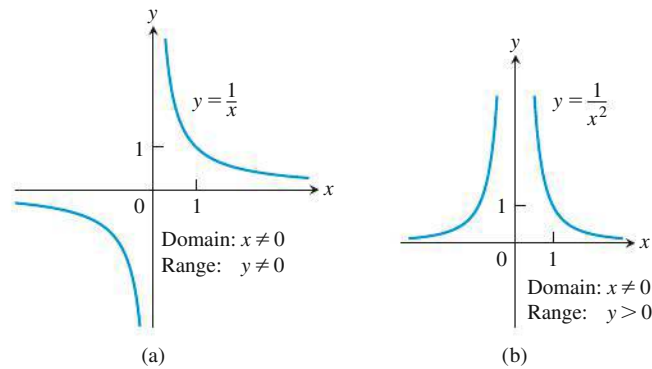


FIGURE 1.16 Graphs of the power functions $f(x) = x^a$.
(a) $a = -1$, (b) $a = -2$.

(c) $a = \frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{2}$, and $\frac{2}{3}$.

The functions $f(x) = x^{1/2} = \sqrt{x}$ and $f(x) = x^{1/3} = \sqrt[3]{x}$ are the **square root** and **cube root** functions, respectively. The domain of the square root function is $[0, \infty)$, but the cube root function is defined for all real x . Their graphs are displayed in Figure 1.17, along with the graphs of $y = x^{3/2}$ and $y = x^{2/3}$. (Recall that $x^{3/2} = (x^{1/2})^3$ and $x^{2/3} = (x^{1/3})^2$.)

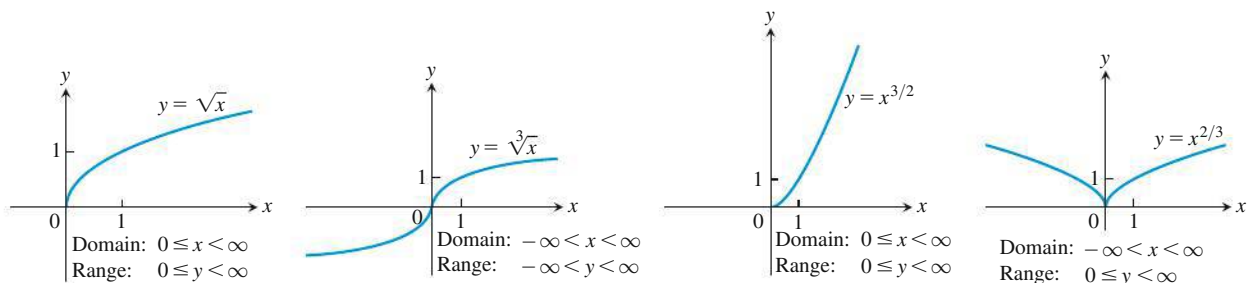


FIGURE 1.17 Graphs of the power functions $f(x) = x^a$ for $a = \frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{2}$, and $\frac{2}{3}$.

Polynomials A function p is a **polynomial** if

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \dots, a_n$ are real constants (called the **coefficients** of the polynomial). All polynomials have domain $(-\infty, \infty)$. If the leading coefficient $a_n \neq 0$, then n is called the **degree** of the polynomial. Linear functions with $m \neq 0$ are polynomials of degree 1. Polynomials of degree 2, usually written as $p(x) = ax^2 + bx + c$, are called **quadratic functions**. Likewise, **cubic functions** are polynomials $p(x) = ax^3 + bx^2 + cx + d$ of degree 3. Figure 1.18 shows the graphs of three polynomials. Techniques to graph polynomials are studied in Chapter 4.

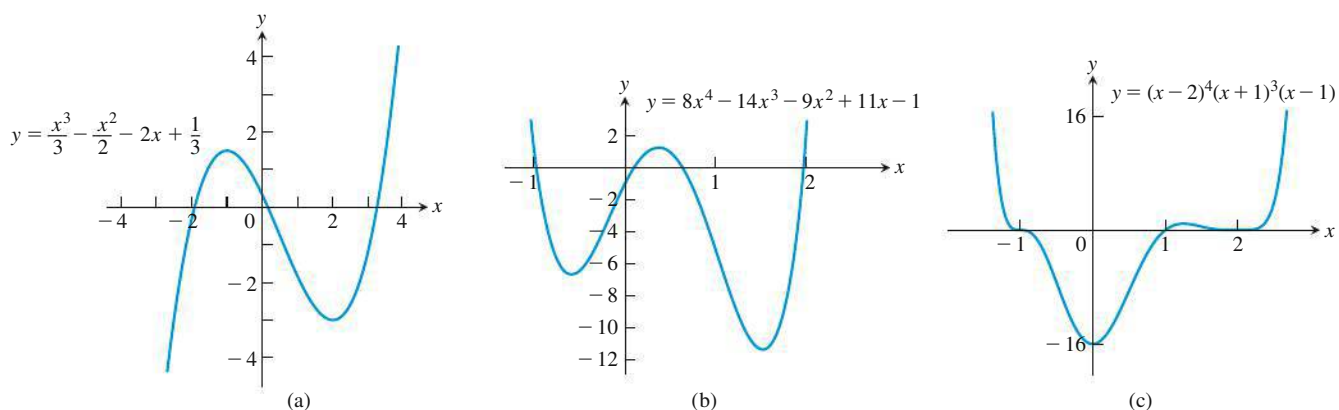


FIGURE 1.18 Graphs of three polynomial functions.

Rational Functions A **rational function** is a quotient or ratio $f(x) = p(x)/q(x)$, where p and q are polynomials. The domain of a rational function is the set of all real x for which $q(x) \neq 0$. The graphs of several rational functions are shown in Figure 1.19.

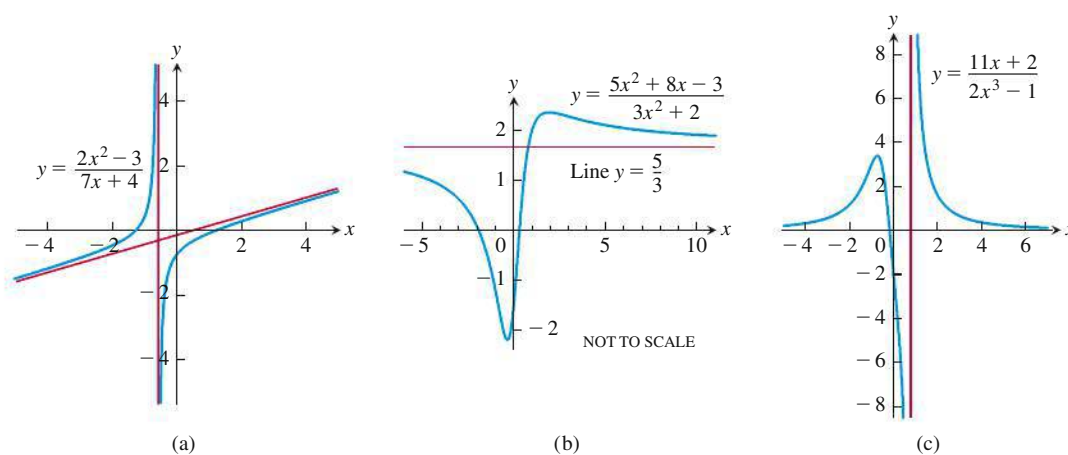


FIGURE 1.19 Graphs of three rational functions. The straight red lines approached by the graphs are called *asymptotes* and are not part of the graphs. We discuss asymptotes in Section 2.6.

Algebraic Functions Any function constructed from polynomials using algebraic operations (addition, subtraction, multiplication, division, and taking roots) lies within the class of **algebraic functions**. All rational functions are algebraic, but also included are more